# Problem 4.5

Show that

$$\Theta(\theta) = A \ln [\tan(\theta/2)]$$

satisfies the $\theta$ equation (Equation 4.25), for $\ell = m = 0$. This is the unacceptable "second solution"—what's *wrong* with it?

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### Solution

Equation 4.25 on page 135 is the ODE for $\Theta(\theta)$.

$$\sin\theta \frac{d}{d\theta}\left(\sin\theta \frac{d\Theta}{d\theta}\right) + \left[\ell(\ell+1)\sin^2\theta - m^2\right]\Theta = 0 \tag{4.25}$$

If $\ell = 0$ and $m = 0$, then

$$\sin\theta \frac{d}{d\theta}\left(\sin\theta \frac{d\Theta}{d\theta}\right) = 0.$$

Divide both sides by $\sin\theta$.

$$\frac{d}{d\theta}\left(\sin\theta \frac{d\Theta}{d\theta}\right) = 0$$

Integrate both sides with respect to $\theta$.

$$\sin\theta \frac{d\Theta}{d\theta} = C_1$$

Divide both sides by $\sin\theta$.

$$\frac{d\Theta}{d\theta} = \frac{C_1}{\sin\theta}$$

Integrate both sides with respect to $\theta$ again.

$$\begin{aligned}\Theta(\theta) &= \int \frac{C_1}{\sin\theta} + C_2 \\ &= C_1 \int \csc\theta + C_2 \\ &= C_1 \ln\left|\tan\frac{\theta}{2}\right| + C_2\end{aligned}$$

And since $0 \leq \theta \leq \pi$, the absolute value sign can be removed.

$$\Theta(\theta) = C_1 \ln\left(\tan\frac{\theta}{2}\right) + C_2$$

The logarithm is the unacceptable second solution. As Mr. Griffiths said on page 136, the problem with it is that it blows up at $\theta = 0$ and $\theta = \pi$. Therefore, it's irrelevant physically.